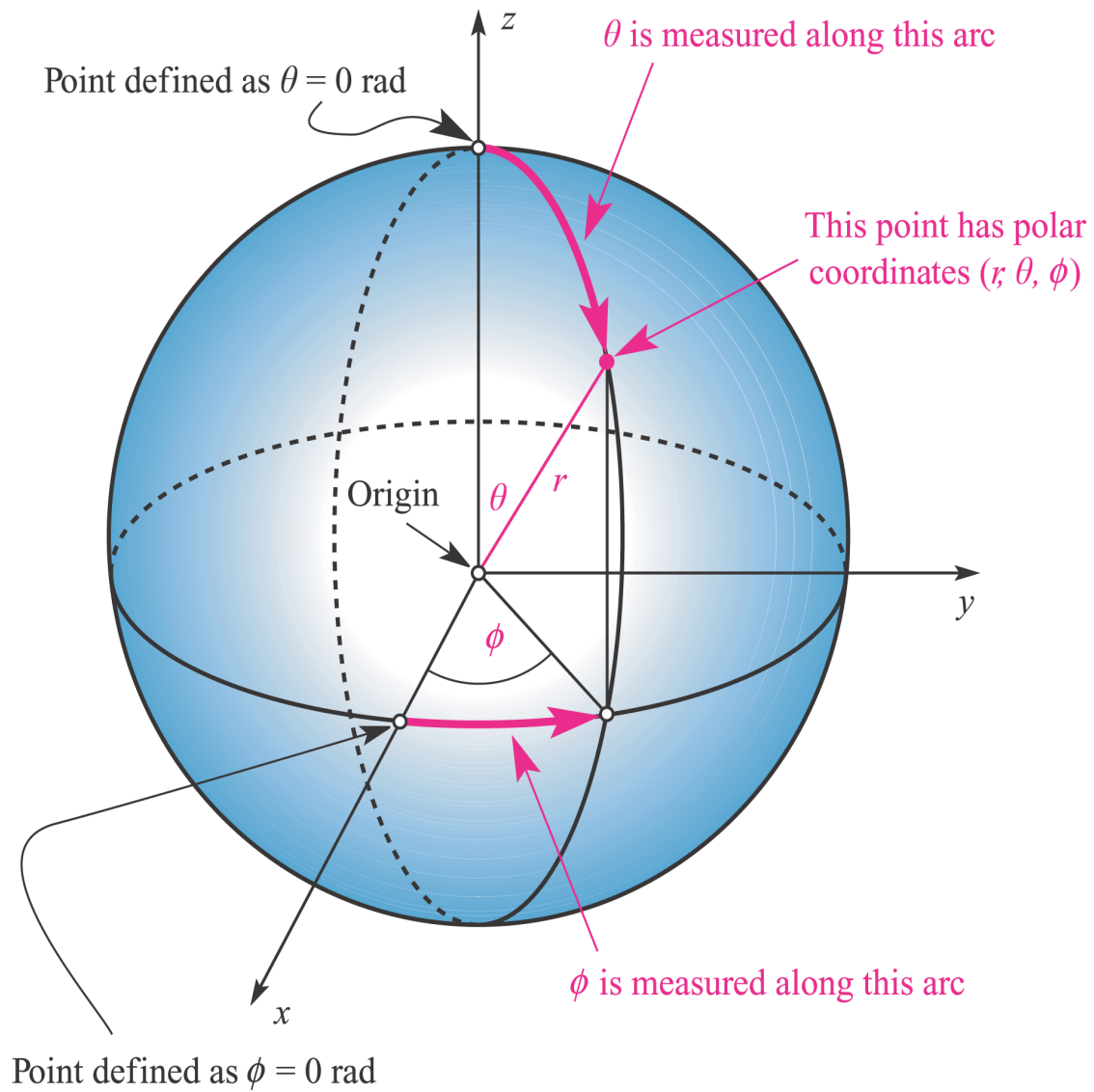




Normalized Wave Functions for Hydrogen Atom

s orbitals

Quantum numbers

$n \quad \ell \quad m_\ell$

$$1 \quad 0 \quad 0 \quad \psi_{1s} = \frac{1}{\sqrt{\pi}} \left(\frac{Z}{a_0} \right)^{3/2} e^{-\frac{Z}{a_0} r}$$

$$2 \quad 0 \quad 0 \quad \psi_{2s} = \frac{1}{4\sqrt{2\pi}} \left(\frac{Z}{a_0} \right)^{3/2} \left(2 - \frac{Z}{a_0} r \right) e^{-\frac{Z}{2a_0} r}$$

$$3 \quad 0 \quad 0 \quad \psi_{3s} = \frac{1}{81\sqrt{3\pi}} \left(\frac{Z}{a_0} \right)^{3/2} \left(27 - \frac{18Z}{a_0} r + \frac{2Z^2}{a_0^2} r^2 \right) e^{-\frac{Z}{3a_0} r}$$

Radial Wave Functions R(r) for Hydrogen Atom

Quantum numbers

$n \quad \ell$

R(r)

$$1 \quad 0 \quad 2 \left(\frac{Z}{a_0} \right)^{3/2} e^{-\frac{Z}{a_0} r}$$

$$2 \quad 0 \quad \left(\frac{1}{2\sqrt{2}} \right) \left(\frac{Z}{a_0} \right)^{3/2} \left(2 - \frac{Z}{a_0} r \right) e^{-\frac{Z}{2a_0} r}$$

$$3 \quad 0 \quad \left(\frac{2}{81\sqrt{3}} \right) \left(\frac{Z}{a_0} \right)^{3/2} \left(27 - \frac{18Z}{a_0} r + \frac{2Z^2}{a_0^2} r^2 \right) e^{-\frac{Z}{3a_0} r}$$

Angular Wave Functions $\Theta\Phi(\Theta\phi)$ for Hydrogen Atom

Quantum numbers

$\ell \quad m_\ell$

$$0 \quad 0 \quad \frac{1}{2\sqrt{\pi}}$$

$$a_0 = (4 \pi \epsilon_0 \hbar^2) / (m_e e^2 Z)$$

Normalized Wave Functions for Hydrogen Atom

p orbitals

Quantum numbers

$n \quad \ell \quad m_\ell$

$$2 \quad 1 \quad 0 \quad \Psi_{2p_z} = \frac{1}{4\sqrt{2\pi}} \left(\frac{Z}{a_0} \right)^{3/2} \frac{Z}{a_0} r e^{-\frac{Z}{2a_0}r} \cos\theta$$

$$2 \quad 1 \quad \pm 1 \quad \Psi_{2p_x, 2p_y} = \frac{1}{4\sqrt{2\pi}} \left(\frac{Z}{a_0} \right)^{3/2} \frac{Z}{a_0} r e^{-\frac{Z}{2a_0}r} \sin\theta \cos\phi$$

$$3 \quad 1 \quad 0 \quad \Psi_{3p_z} = \frac{\sqrt{2}}{81\sqrt{\pi}} \left(\frac{Z}{a_0} \right)^{3/2} \left(\frac{6Z}{a_0} r - \frac{Z^2}{a_0^2} r^2 \right) e^{-\frac{Z}{3a_0}r} \cos\theta$$

$$3 \quad 1 \quad \pm 1 \quad \Psi_{3p_x, p_y} = \frac{\sqrt{2}}{81\sqrt{\pi}} \left(\frac{Z}{a_0} \right)^{3/2} \left(\frac{6Z}{a_0} r - \frac{Z^2}{a_0^2} r^2 \right) e^{-\frac{Z}{3a_0}r} \sin\theta \cos\phi$$

Radial Wave Functions R(r) for Hydrogen Atom

Quantum numbers

$n \quad \ell$

$$2 \quad 1 \quad R(r) = \left(\frac{1}{2\sqrt{6}} \right) \left(\frac{Z}{a_0} \right)^{3/2} \frac{Z}{a_0} r e^{-\frac{Z}{2a_0}r}$$

$$3 \quad 1 \quad \left(\frac{4}{81\sqrt{6}} \right) \left(\frac{Z}{a_0} \right)^{3/2} \left(\frac{6Z}{a_0} r - \frac{Z^2}{a_0^2} r^2 \right) e^{-\frac{Z}{3a_0}r}$$

Angular Wave Functions $\Theta\Phi(\Theta\phi)$ for Hydrogen Atom

Quantum numbers

$\ell \quad m_\ell$

$$1 \quad 0 \quad \frac{1}{2} \sqrt{\frac{3}{\pi}} \cos\theta$$

$$\pm 1 \quad \frac{1}{2} \sqrt{\frac{3}{\pi}} (\sin\theta \cos\phi)$$

$$a_0 = (4 \pi \epsilon_0 \hbar^2) / (m_e e^2 Z)$$

Normalized Wave Functions for Hydrogen Atom

d orbitals

Quantum numbers

$n \quad \ell \quad m_\ell$

$$3 \quad 2 \quad 0 \quad \Psi_{3d_{z^2}} = \frac{1}{81\sqrt{6\pi}} \left(\frac{Z}{a_0} \right)^{3/2} \frac{Z^2}{a_0^2} r^2 e^{-\frac{Z}{3a_0}r} (3\cos^2\theta - 1)$$

$$3 \quad 2 \quad \pm 1 \quad \Psi_{3d_{xz}, 3d_{yz}} = \frac{\sqrt{2}}{81\sqrt{\pi}} \left(\frac{Z}{a_0} \right)^{3/2} \frac{Z^2}{a_0^2} r^2 e^{-\frac{Z}{3a_0}r} \sin\theta \cos\theta \cos\phi$$

$$3 \quad 2 \quad \pm 2 \quad \Psi_{3d_{xy}, 3d_{x^2-y^2}} = \frac{1}{81\sqrt{2\pi}} \left(\frac{Z}{a_0} \right)^{3/2} \frac{Z^2}{a_0^2} r^2 e^{-\frac{Z}{3a_0}r} \sin^2\theta \cos 2\phi$$

Radial Wave Functions R(r) for Hydrogen Atom

Quantum numbers

$n \quad \ell$

R(r)

$$3 \quad 2 \quad \left(\frac{4}{81\sqrt{30}} \right) \left(\frac{Z}{a_0} \right)^{3/2} \frac{Z^2}{a_0^2} r^2 e^{-\frac{Z}{3a_0}r}$$

Angular Wave Functions $\Theta\Phi(\Theta\phi)$ for Hydrogen Atom

Quantum numbers

$\ell \quad m_\ell$

$$2 \quad 0 \quad \frac{1}{4} \sqrt{\frac{5}{\pi}} (3\cos^2\theta - 1)$$

$$\pm 1 \quad \frac{1}{2} \sqrt{\frac{15}{\pi}} \cos\theta \sin\theta \cos\phi$$

$$\pm 2 \quad \frac{1}{4} \sqrt{\frac{15}{\pi}} \sin^2\theta \cos 2\phi$$

$$a_0 = (4\pi\epsilon_0\hbar^2) / (m_e e^2 Z)$$

